Magnetic Compensation of a Gravity-Gradient Stabilized Satellite

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The attitude control, using environmental magnetic torques, of a gravity-stabilized satellite, damped with control moment gyros, is investigated. By observing vehicle response on-orbit, the actual environmental torques are evaluated, and onboard magnets are adjusted, using quadratic error criteria, to offset external torques and produce minimum attitude librations. Flight data and relevant simulated responses are presented to verify modeling accuracy and to illustrate the effectiveness of the compensation technique. For the cases considered, compensating magnetic moments are shown to produce significant reduction in satellite motion over a 30 orbit period for a range of satellite altitudes.

Nomenclature

a	= area moment
A	= direction cosine matrix relating the body frame to the orbital frame
A_{ij}	= element of A in ith row and jth column
$\mathbf{a}_n,\mathbf{b}_n$	= coefficients of modified Fourier series
c	= abbreviation for cosine
D_i	= viscous gain of CMG$_i$
e	= eccentricity of the orbit
H	= total angular momentum of the control moment gyros (CMG's)
H_B	= angular momentum of the satellite rigid body
H_i	= angular momentum of CMG$_i$
h_i	= magnitude of angular momentum of CMG$_i$
I	= satellite rigid body moment of inertia tensor
I_x,I_y,I_z	= principal moments of inertia
I_{xy},I_{xz},I_{yz}	= products of inertia
i	= orbit inclination
k	= Earth's gravity constant
K_i	= spring constant of CMG$_i$
M	= external torque exclusive of gravity torque
M_G	= gravity-gradient torque
M_s	= additive solar torque
M_T	= total external torque
$\mathbf{O}_i$	= unit vector representing the output axis of CMG$_i$
P	= desired perturbation of spacecraft parameter
$\hat{P}$	= nominal perturbation of spacecraft parameter
$\Delta P(m)$	= desired perturbation of magnetic moment
$\Delta\hat{P}(m)$	= nominal perturbation of magnetic moment
Q	= transformation matrix relating modified Fourier coefficients to flight data
$\mathbf{R}$	= unit radius vector
r	= magnitude of radius vector
s	= abbreviation for sine
t	= time
T	= matrix transpose
$\mathbf{x}$	= modified Fourier series coefficient vector
$\mathbf{x}_0$	= coefficient vector due to eccentricity
x,y,z	= body frame vector components
$\mathbf{x}_{ax},\mathbf{x}_{ay},\mathbf{x}_{az}$	= coefficient vectors due to area moments
$\mathbf{x}_{mx},\mathbf{x}_{my},\mathbf{x}_{mz}$	= coefficient vectors due to magnetic moments
X	= matrix composed of vectors $\mathbf{x}_j$
$\mathbf{y}$	= flight data time history vector
$\boldsymbol{\alpha}$	= perturbation factor vector
β	= CMG vee angle—one half the included angle between the spin reference axes of a pair of CMG's
$\dot{\mu}$	= true anomaly rate
σ_i,φ_i	= gimbal angle of CMG$_i$, $\beta - \sigma_i$, respectively
$\dot{\sigma}_i$	= gimbal angular speed of CMG$_i$
τ_i	= gimbal torque due to torque generator and drift on CMG$_i$
ψ	= yaw angle
$\tilde{\psi}$	= oscillatory component of yaw angle
ω	= angular velocity of rigid body
ω_0	= mean anomaly rate $\omega_0 = 2\pi$/period
ω_D	= relative angular velocity between body frame and orbital frame
ω_R	= angular velocity of orbital reference frame
(γx)	= cross product matrix of dummy vector $\boldsymbol{\gamma}$
$(\dot{\ })$	= time derivative of ( )
$(\ )^i$	= quantity ( ) with reference to inertial frame
$(\ )^b$	= quantity ( ) with reference to body frame

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Introduction

THE attitude performance of a gravity-stabilized spacecraft depends on the external torques acting on it. These torques arise primarily from solar and aerodynamic pressures, orbit eccentricity, and the tendency of residual magnetic moments to align with the Earth's magnetic field. For medium altitude, near circular satellite orbits, residual magnetic moments interacting with the Earth's magnetic field often produce the dominant disturbance torques. In these cases, close magnetic moment control can result in significantly improved attitude accuracy.

Control procedures are normally established in the design and manufacture of gravity-stabilized spacecraft to minimize magnetic moments. However, unless extreme care is taken, a relatively large magnetic moment usually remains after assembly. Measurements can be made of the vehicle magnetic field to deduce the magnetic moments; however, the accuracy of the measurements tends to decrease with increasing size of the spacecraft. In addition, ground determination of the moments is of limited value since it has been observed that the magnetic moments often change between test and orbit operation. Degaussing is also not always possible due to the type of materials and equipment used in the spacecraft. As a result, only the large magnetic moments due to specific components are usually compensated by

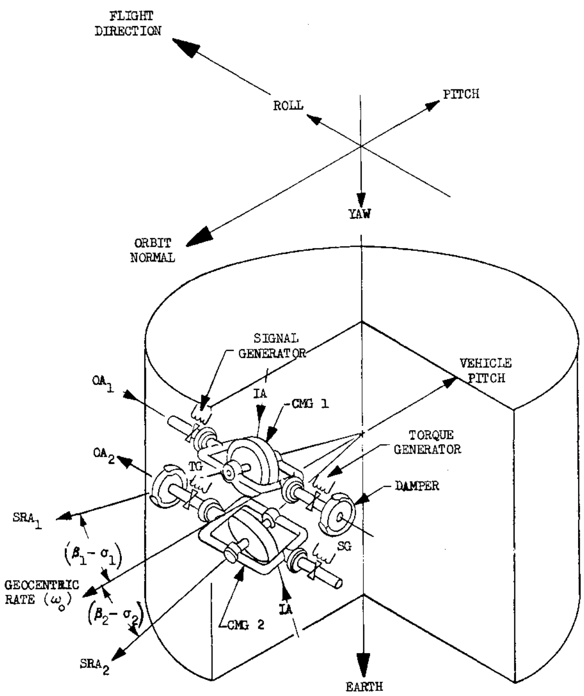

Fig. 1 Roll-skewed VCMG system orientation.

opposing magnets. Hence, attitude accuracy has been limited by the remaining residual magnetic moments.

The purpose of the present paper is to describe a relatively straightforward method for determining magnetic moments which may be ground commanded to minimize vehicle attitude excursions. Using the on-orbit compensation technique, the need for accurate pre-flight magnet moment determination is avoided, and the capability for identifying and countering the effects of other external torques is provided. Although this technique is applicable to a wide range of gravity stabilized vehicles, an Agena spacecraft, using control moment gyros (CMG) for semipassive damping, will be considered.

Previous work by Farrell et al.[1] and by Tinling and Merrick[2] considered the problem of estimating the attitude motion of satellites which employed flexible booms for gravity stabilization. Although the mathematical modeling of Farrell is, in some ways, quite complex, the techniques used in both papers are similar to those given here. However, modeling accuracies are not verified by processing actual flight data and methods of reducing attitude librations by adjusting system parameters on-orbit are not discussed in these papers.

Compensation Technique

The compensation technique is basically a least-squared error determination of the unknown spacecraft parameters, including magnetic moments, from one orbit of observed attitude data. This knowledge is used to estimate the values of magnetic moments required to minimize attitude motion. These desired values are then commanded to the vehicle.

Estimation of unknown parameters requires a model, in this case implemented by a digital computer simulation, of the gravity-stabilized satellite and the environmental disturbances due to magnetic, solar, and aerodynamic interactions. Usually, such parameters as c.m., CMG angular momentum, and vehicle moments of inertia are accurately known. Also, intensities of external disturbances such as solar pressure, aerodynamic effects, and magnetic fields can be calculated for particular orbits of interest. What is unknown about the system are those parameters describing the interaction of the vehicle with the environment, e.g., area and magnetic moments, and drag coefficients. Once the satellite is placed in orbit, however, observed vehicle motion can be used, together with the simulation, to estimate these unknown parameters. Since attitude motion for this type of satellite is a somewhat periodic forced oscillation produced by complex environmental disturbance torques,[3] a digital simulation provides an appropriate solution.

The following operational sequence is employed: 1) flight attitude data is processed to remove telemetry channel noise; 2) equivalent on-orbit gyro parameters and sun sensor yaw values are computed closed form; 3) area moments and magnetic moments are estimated from perturbed vehicle responses using a minimum squared error criterion; and 4) the changes in spacecraft magnetic moments necessary to minimize attitude excursions are determined. Relevant details of the last three areas are discussed below, preceded by a description of the system model.

System Modeling

Since the magnetic compensation technique depends on an accurate simulation model, some characteristics of the disturbance environment and a brief description of the nonlinear vehicle dynamics are given for later reference. A similar derivation of the equations of motion has been given in Refs. 3 and 4.

Magnetic, aerodynamic, and solar environmental disturbances were incorporated into the simulation. The Earth's magnetic field was represented for computational efficiency by a twelve offset dipole approximation to 1960 through 1970 Earth field data.[5] The fitting error was less than 2% for low-altitude orbits. Aerodynamic torques were accounted for by a Harris-Priester atmospheric model[6] incorporating solar bulge effects. Satellite position and relevant sun orientation were given by sun and satellite ephemerides including second order Earth oblateness terms. Solar pressure disturbances were computed from structural models incorporated as part of the simulation. Frequently, however, uncertainties in the reflectance and specularity of structural components required that additional solar torque components be added "open-loop."

The gravity-stabilized Agena spacecraft includes control moment gyros in a vee configuration to provide damping and improve yaw stiffness. This system is described in detail by numerous references, recently in Ref. 3. As shown in Fig. 1, the spin vectors of the gyros form a vee in the vertical plane with the pitch axis bisecting the vee. The resulting angular momentum vector is nominally aligned to the orbital angular velocity vector. Both gyros are electrically torqued to maintain the vee and prevent alignment of orbit rate and gyro angular momenta. Scissoring motion of the CMG's provides pitch damping, while rotation in the same direction produces yaw damping and, by roll-yaw cross coupling, damping in the roll axis. The system equations of motion are now briefly developed in matrix form.

From Euler's equation of motion $M_T = M + M_G = (\dot{H}_B)^i + (\dot{H})^i$ and

$$M = I\dot{\omega} + (\omega x)I\omega + (\dot{H})^b + (\omega x)H - 3(k/r^3)(Rx)IR \quad (1)$$

The kinematics of attitude motion will be specified in terms of the direction cosine matrix A, rather than some of the more kinematic representations (i.e., Euler Angles, Euler parameters, Gibbs parameters, etc.) in order to minimize notation and geometric developments. The direction cosine matrix A relates the body frame relative to an orbit reference frame. The body frame is defined as (X,Y,Z) where X is the roll axis, Y is the pitch axis and Z is the yaw axis. The orbit reference frame (X_0,Y_0,Z_0) is established by the unit velocity vector (for a circular orbit), unit negative orbital angular velocity vector, and unit gravity vector, respectively. The time derivative of A can be determined from Coriolis's law as

$$\dot{A} = -(\omega_D x)A \quad (2)$$

$$\dot{A} = -(\omega x)A + A(\omega_R x) \quad (3)$$

Equation (3) results from the similarity transformation of A on the cross-product matrix $(\omega_R x)$ coordinatized in the body frame rather than the orbit frame.

With the body frame selected to coincide with the satellite's principal axes, the moment of inertia matrix is diagonal with elements I_x,I_y,I_z, so that Eq. (1) may be written as

$$M_x = I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z + \dot{H}_x + \omega_y H_z - \omega_z H_y + (3k/r^3)(I_y - I_z)A_{23}A_{33} \quad (4)$$

$$M_y = I_y\dot{\omega}_y + (I_x - I_z)\omega_x\omega_z + \dot{H}_y + \omega_z H_x - \omega_x H_z - (3k/r^3)(I_x - I_z)A_{13}A_{33} \quad (5)$$

$$M_z = I_z\dot{\omega}_z + (I_y - I_x)\omega_x\omega_y + \dot{H}_z + \omega_x H_y - \omega_y H_x - (3k/r^3)(I_y - I_x)A_{13}A_{23} \quad (6)$$

If the orbital motion is that of the undisturbed central force field two-body problem, where the Earth's oblateness, atmospheric drag, and sun-moon effects are ignored, the angular velocity of the orbital reference frame becomes

$$\omega_D = -\begin{bmatrix} 0 \\ \dot{\mu} \\ 0 \end{bmatrix} \quad (7)$$

where

$$\dot{\mu} = [\omega_0/(1 - e^2)^{3/2}](1 + ec\mu)^2 \cong \omega_0(1 + 2ec\omega_0 t) \quad (8)$$

and

$$k/r^3 = \dot{\mu}^2/(1 + ec\mu) \cong \omega_0^2(1 + 3ec\omega_0 t) \quad (9)$$

The CMG dynamics may also be derived from Euler's equation of motion. Since the CMG is constrained to rotate only about its output axis, its dynamic equation is scalar. The effect of the polar moment of inertia of the gyro's gimbal can be neglected since the normal gimbal time constant is of the order of milliseconds while the vehicle time constant is of the order of kiloseconds. The equation of motion for the ith CMG (CMG_i) is

$$D_i\dot{\sigma}_i + K_i\sigma_i = \tau_i - O_i^T(\omega x)H_i \quad (10)$$

Equation (10) may now be written explicitly for the roll-skewed VCMG system. For this system, the X component of H is zero, since the output axes of each gyro lie along the roll or X axis. Assume that both gyros are oriented symmetrically above and below the minus Y axis, in the vertical plane, by the angle β, CMG_1 has its output axis along the negative X axis with its spin axis above the horizontal plane, and CMG_2 has its output axis along the positive X axis with its spin axis below the horizontal plane (Fig. 1). The angular momentum of each CMG is, then, letting $\varphi_i = \beta - \sigma_i$

$$H_1 = h_1\begin{bmatrix} 0 \\ -c(\varphi_1) \\ -s(\varphi_1) \end{bmatrix} \qquad H_2 = h_2\begin{bmatrix} 0 \\ -c(\varphi_2) \\ +s(\varphi_2) \end{bmatrix} \quad (11)$$

Substituting Eq. (11) into Eqs. (4–6) yields the rigid body equations, cf. Refs. 3 and 4, and

$$D_i\dot{\sigma}_i + K_i\sigma_i = \tau_i - \omega_y h s(\varphi_i) \pm \omega_z h c(\varphi_i) \quad (12)$$

$$\tau_i = -\omega_0 h s\beta; \quad i = 1,2 \quad (13)$$

where the electrical torque generator commands on each CMG, τ_i, are adjusted to cause the gyro gimbal to operate about its null. With these equations, some preliminary flight data processing may be accomplished.

Estimation of Gyro Parameters

Before evaluation of the interaction of the spacecraft and the environment can be accomplished, modeling for the control system must be made more precise. Although the structural properties of the vehicle are usually known quite accurately prior to launch, the gyro parameters are subject to large uncertainty since accurate measurements in Earth gravity are not feasible for this type of gyro. Therefore, it is advantageous to calculate the damping coefficients and torque generator biases for the CMGs installed in the spacecraft. The calculation proceeds by eliminating ω_z from Eq. (12), which yields, for zero spring constant K_i

$$[\dot{\sigma}_1 c(\varphi_2),\ \dot{\sigma}_2 c(\varphi_1),\ -c(\varphi_2),\ -c(\varphi_1)]\begin{bmatrix} D_1 \\ D_2 \\ \tau_1 \\ \tau_2 \end{bmatrix} = \omega_y h s(\sigma_1 - \sigma_2) \quad (14)$$

Evaluating Eq. (14) at n discrete times, letting

$$F \triangleq \begin{bmatrix} \dot{\sigma}_1(t_1)c[\varphi_2(t_1)], & \dot{\sigma}_2(t_1)c[\varphi_1(t_1)], & -c[\varphi_2(t_1)], & -c[\varphi_1(t_1)] \\ \vdots & \vdots & \vdots & \vdots \\ \dot{\sigma}_1(t_n)c[\varphi_2(t_n)], & \dot{\sigma}_2(t_n)c[\varphi_1(t_n)], & -c[\varphi_2(t_n)], & -c[\varphi_1(t_n)] \end{bmatrix} \quad (15)$$

and weighting the parameters equally, Eq. (14) is inverted, for minimum squared error[7] by

$$\begin{bmatrix} D_1 \\ D_2 \\ \tau_1 \\ \tau_2 \end{bmatrix} = (F^T F)^{-1} F^T \begin{bmatrix} \omega_y(t_1)s[\sigma_1(t_1) - \sigma_2(t_1)] \\ \vdots \\ \omega_y(t_n)s[\sigma_1(t_n) - \sigma_2(t_n)] \end{bmatrix} \quad (16)$$

All quantities except $[D_1,D_2,\tau_1,\tau_2,]^T$ are known functions of time. Pitch angular velocity ω_y is deduced from processed horizon sensor data and the ephemeris. A unique solution for the unknown parameters is obtained when Eq. (14) is evaluated at four or more discrete times. With gyro characteristics determined, Eq. (12) can be used to compute ω_z, whence yaw follows by integration. The integration constant is not known, however, so that the yaw obtained is only the oscillatory component, denoted $\tilde{\psi}$. It can be shown, in fact, that yaw motion is not observable from just horizon sensor and gyro gimbal data. The only method for eliminating the bias error from $\tilde{\psi}$, and hence, determine system yaw, is to provide independent yaw measurement at discrete times throughout the orbit. A sun sensor can be employed for this purpose.

Both $\tilde{\psi}$ and gyro parameters are subject to errors in CMG modeling. Deviations from an ideal gyro such as spring restraint and gimbal stiction effects could introduce appreciable errors into the calculation. Furthermore, the gyros used for this particular system are known to possess strongly temperature dependent damping coefficients D_i. The solution (16) then, must be considered to describe average system parameters and not to give point-wise evaluation of gyro performance.

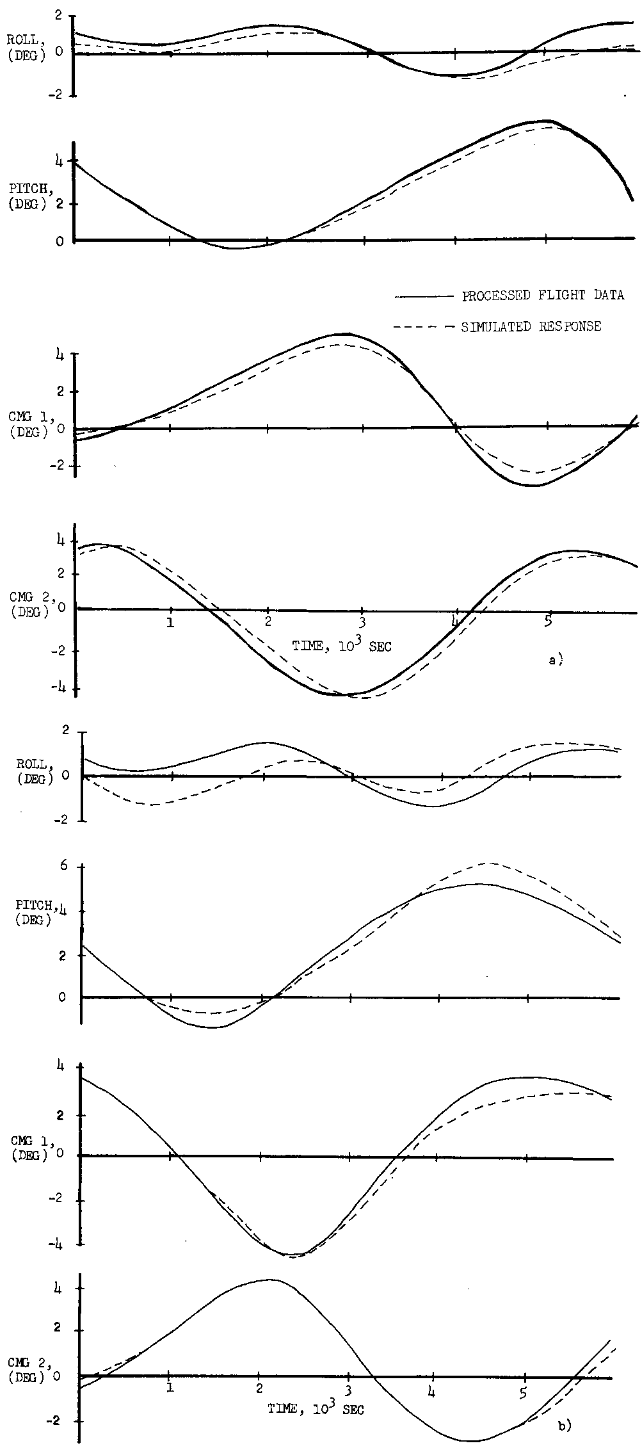

Fig. 2 Comparison of flight data and simulated response for Agena satellite "A": a) initial orbit, b) 120 orbits later.

Estimation of Spacecraft Parameters

Once the characteristics of the control moment gyros are obtained, it remains to describe the interaction of the disturbance environment with the satellite. The effects of solar pressure, aerodynamic drag, and the Earth's magnetic field must be considered. Thus, area moments and magnetic moments must be identified for each vehicle. If solar pressure disturbances are not accounted for directly in the dynamic model, additional external torques may have to be determined. The basic perturbation technique for accomplishing these tasks will now be described.

Flight data for the orbiting satellite giving vehicle roll, pitch, and CMG gimbal angles as functions of time over one complete orbit is obtained. For convenience in handling this data, these time functions are represented by a modified, truncated Fourier series such that

$$\mathbf{y}(t) = \mathbf{a}_0 + \sum_{n=1}^{5} [\mathbf{a}_n \cos n\omega_0 t + \mathbf{b}_n \sin n\omega_0 t] + \frac{\mathbf{a}_6 \omega_0 t}{2\pi} \quad (17)$$

where $\mathbf{y}(t)$ is the flight data vector and t is time, usually referenced to the ascending node. The components of $\mathbf{y}$ include roll, pitch, and CMG gimbal angles, along with processed sun sensor yaw data (if available). The calculation is affected over a set of times $[t_j]_{j=1,N}$ by rewriting Eq. (17), viz.,

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}(t_1) \\ \mathbf{y}(t_2) \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{y}(t_N) \end{bmatrix} = Q\mathbf{x} \quad (18)$$

where $\mathbf{x}$ is the vector of coefficients a_n, b_n, from Eq. (17), and the columns of Q consist of a constant (unity) and the sine, cosine, and ramp functions evaluated at the discrete times t_i. Inverting Eq. (18) yields

$$\mathbf{x} = (Q^T Q)^{-1} Q^T \mathbf{y} \quad (19)$$

for an unweighted minimum squared error.[7] The satellite motion due to disturbances can now be evaluated.

Perturbations to satellite motion are found using a dynamical simulation of the nonlinear equations of motion. System responses, over the orbit represented by the flight data, are obtained separately for each disturbance and converted to a set of modified Fourier coefficients using Eq. (19). Let the coefficients for the basic librations be given by $\mathbf{x}_0$, for eccentricity; by $\mathbf{x}_{ax}, \mathbf{x}_{ay}, \mathbf{x}_{az}$, for area moments; and by $\mathbf{x}_{mx}, \mathbf{x}_{my}, \mathbf{x}_{mz}$, for magnetic moments. These coefficients, which will be referred to generally as $\mathbf{x}_j$, are computed from Eq. (19), where the time history vectors $\mathbf{y}$ result from simulation runs with nominal perturbations of magnitude $\hat{P}_j$ of the parameters of interest (e.g., area moments, magnetic moments). If the system is assumed linear, it is then possible to write

$$\mathbf{x} - \mathbf{x}_0 = \sum_{j=1}^{M} \alpha_j \mathbf{x}_j \quad (20)$$

where the α's indicate the contribution of each of the perturbation motions to the over-all response.

Using a quadratic error criterion, Eq. (20) is solved in the presence of noise and model inaccuracies by

$$\alpha = (X^T X)^{-1} X^T (\mathbf{x} - \mathbf{x}_0) \quad (21)$$

where the columns of the matrix X are the vectors $\mathbf{x}_j$. Hence, the desired vehicle parameters P_j are given by

$$P_j = \alpha_j \hat{P}_j \quad (22)$$

These parameters are set in the simulation and the vehicle response to all the perturbations is observed. If this response matches flight data within allowable limits, modeling is complete and estimation and prediction of attitude motion is accomplished by running the simulation. If not, this

Table 1 Physical characteristics of Agena flight test vehicle A

CMG:	Mod I, Kearfott
I_x, I_y, I_z	= 6000, 6000, 600, ft-lb-sec²
H	= 3.32, D = 2.21, ft-lb-sec
β	= 20°, ω_0 = 0.0011 rad/sec, i = 75°

motion, generated by P_j, is used as a new $\mathbf{x}_0$, and the procedure using Eq. (20) is repeated until the simulated and actual motions agree to within an acceptable error bound. The perturbed responses, $\mathbf{x}_j$, are now obtained for these iterations by incrementing the currently calculated value of P_j as obtained from Eq. (22). Actual processing has shown that more than one or two iterations produce little improvement in the fit.

Modeling Accuracy

The flight data results given below include evaluation of the fitting and simulation for a 120 orbit time span. Approximate vehicle parameters are given in Table 1. Figure 2a shows actual time histories of an Agena satellite obtained by recording horizon sensor and CMG gimbal outputs over one full orbit. The simulated response, based on fitting vehicle parameters to this flight data is shown. A comparison 120 orbits later is shown in Fig. 2b where simulated response in this case is based on the parameters determined 120 orbits earlier. It can be seen that the actual and simulated responses match very closely. From this result, it may be concluded that the modeling and evaluation of system parameters is accurate. Further proof of the validity of the predictions has been found in other missions where accurate predictions several hundreds of orbits ahead have been made. This prediction capability forms the basis of the extension to the magnetic compensation procedure which is now discussed.

Determination of Compensating Magnetic Moments

The fact that attitude motion can be accurately predicted many orbits ahead implies that vehicle parameters remain fairly constant. Therefore, adjusting the magnetic moments to desired values with one command will improve performance for a relatively long period of time. It remains to determine what values will best improve performance. One choice is simply the negative of the actual magnetic moments as estimated by the flight data fit. A second choice takes advantage of the fact that, for an inclined orbit, a near sinusoidal pitch torque, at orbital frequency and with arbitrary phase angle, can be created by adjusting the magnetic moment components along any two orthogonal axes in the orbit plane. Thus, since the predominant disturbances due to solar and aerodynamics torques and eccentricity occur at orbital frequency, pitch performance can be substantially improved by proper magnetic moment control. The magnetic moments required to minimize attitude excursions may be determined in a manner similar to the fitting procedure. This procedure yields, for the on-orbit spacecraft, a set of parameter values P_j, and attitude librations represented by the coefficients $\mathbf{x}_n$. It is desired to determine the changes in magnetic moments, $\Delta P_j(m)$, which minimize

Table 2 Physical characteristics of typical satellite selected for magnetic compensation evaluation

I_x, I_y, I_z	$= 7300, 8300, 1400$, ft-lb-sec^2
$h_{1,2}$	$= 3.32$, ft-lb-sec
$D_{1,2}$	$= 1.8$ ft-lb-sec
$\beta_{1,2}$	$= 30°$
$m_{x,y,z}$	$= 0.0004$ ft-lb/oersted $= 5423$ UPC
i	$= 90°$
a_x, a_y, a_z	$= 1000, 60, 600$, ft^3
M_{s_x}	$= [1.2 - 15c\omega_0 t - 3c2\omega_0 t - 0.5c3\omega_0 t - 5s\omega_0 t + 1s2\omega_0 t] \times 10^{-5}$ ft-lb
M_{s_y}	$= [-2 - 1.5c\omega_0 t - 10c2\omega_0 t + 0.8c3\omega_0 t + 12s\omega_0 t - 2\sin 2\omega_0 t - 6s2\omega_0 t] \times 10^{-5}$ ft-lb
M_{s_z}	$= [0.8 + 3c\omega_0 t - 0.2\, c3\omega_0 t - 3s\omega_0 t + 0.5\, s2\omega_0 t] \times 10^{-5}$ ft-lb

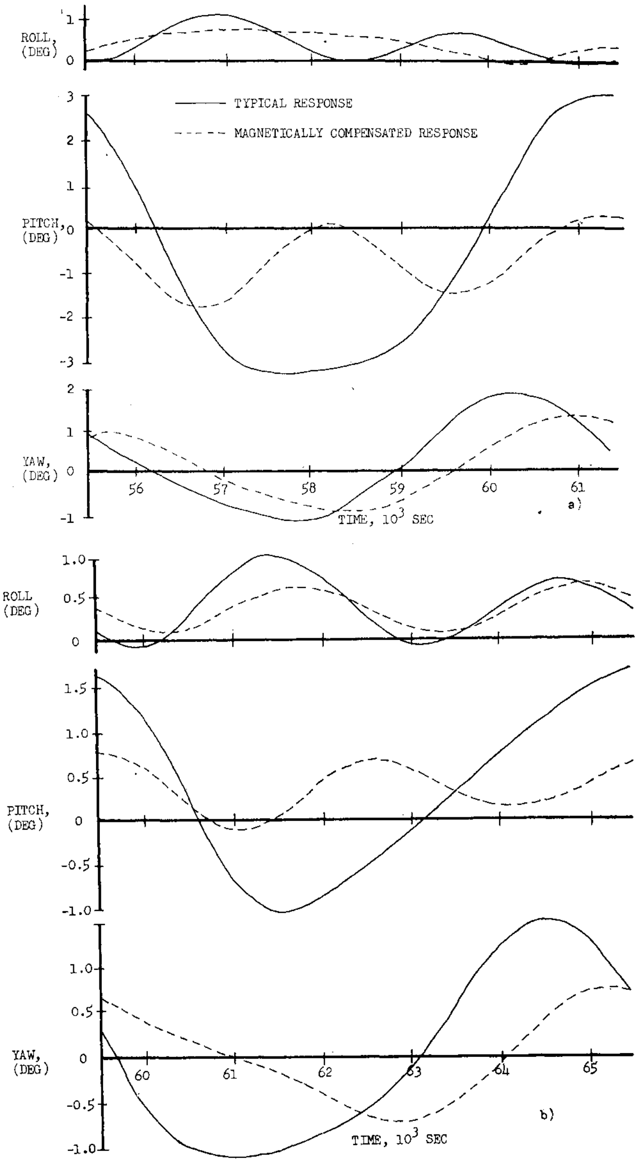

Fig. 3 Comparison of typical Agena satellite simulated responses with and without magnetic compensation: a) 300 naut miles, b) 500 naut miles.

attitude librations. The motions $\mathbf{x}_j(m)$ due to nominal perturbations of the magnetic moments $\Delta\hat{P}_j(m)$ are obtained as before. Hence, for minimum squared error,

$$0 = \mathbf{x}_n + \sum_{j=1}^{3} \alpha_j(m)\mathbf{x}_j(m) \tag{23}$$

Using Eq. (21) to obtain $\boldsymbol{\alpha}(m)$, the desired magnetic moments are

$$\Delta P_j(m) = \alpha_j(m)\Delta\hat{P}_j(m) \tag{24}$$

Many possible methods exist for implementing magnetic moment adjustment on-orbit. Electromagnets, for example, are simple to mechanize but require continuous power dissipation. A fixed array of permanent magnets mounted in a movable fixture is also feasible. A third means is the use of variable permanent magnets surrounded by a magnetizing coil. Power is used only to change the magnetic state of the core, and no moving parts are required. Specific equipment selection would, of course, have to be based on desired system lifetime and over-all cost.

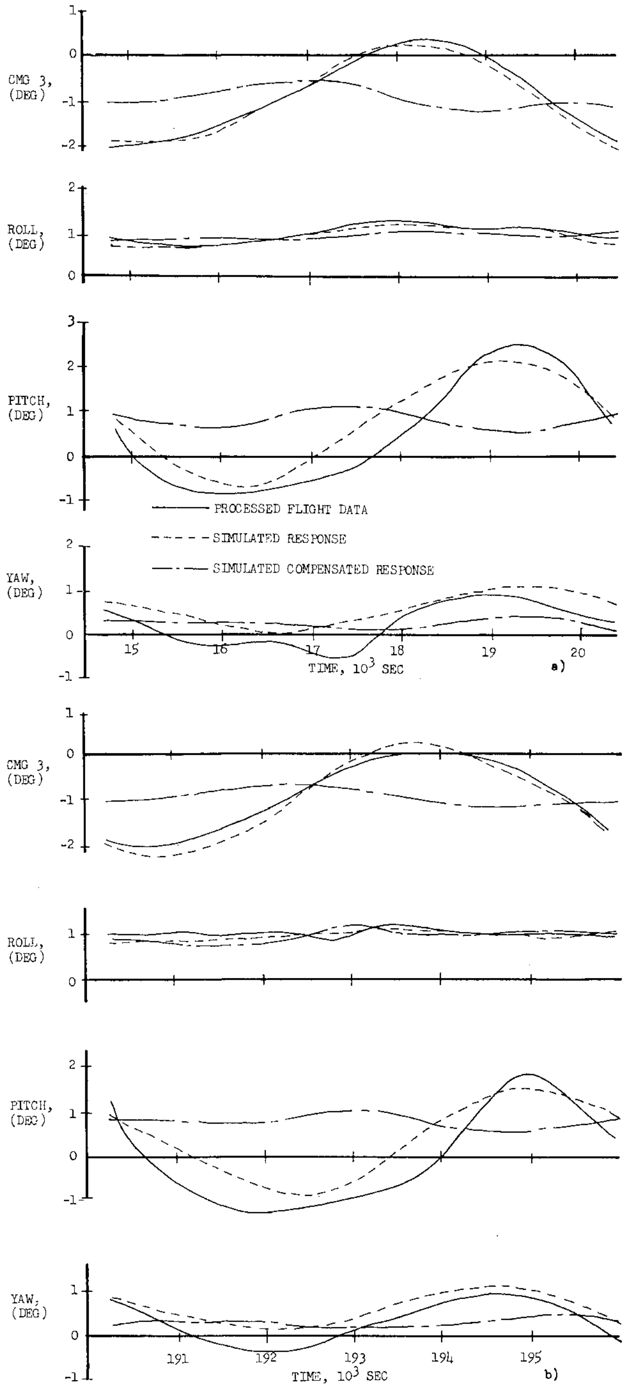

Fig. 4 Comparison of flight data and simulated responses for Agena satellite "B" with and without compensation: a) orbit 742, b) orbit 773.

Magnetic Compensation Performance

To demonstrate the improvements in performance which can be expected from the magnetic compensation procedure, several representative cases were run on the computer. Typical values for the CMG damping and torque generator output, solar torques, aerodynamic area moments, etc., were selected (Table 2). In actual practice, these values would be determined in the manner described above. Nominal values for each of the three components of the residual magnetic moments were taken to be approximately 5000 unit pole centimeters (UPC). Orbit eccentricity was set at 0.005.

Figures 3a and 3b show the results of simulations run at altitude of 300 and 500 naut miles, respectively. Each plot contains two curves corresponding to the nominal simulation and to the simulation which results from using the optimal magnetic moments. The greatest improvements occur in pitch although reductions in roll and yaw also result. This is due to the fact that for an inclined orbit and constant magnetic moments, near-sinusoidal torques at orbit rate occur in all three axes with daily varying biases occurring in the roll and yaw axes. However, the phase angle of the pitch torque may be arbitrarily established by suitable selection of the in-plane magnetic moments. Therefore, the orbit rate libration in pitch may be nearly eliminated. However, pitch bias errors cannot be reduced by magnetic compensation. The situation in roll and yaw is different since normally only that part of the oscillation attributable to the magnetic moments may be removed.

Table 3 Physical characteristics of Agena flight vehicle B

I_x, I_y, I_z	$= 7900, 8200, 1000$, ft-lb-sec^2
I_{xy}, I_{xz}, I_{yz}	$= 100, 200, 260$, ft-lb-sec^2
$h_{1,2,3,4}$	$= 3.32$ ft-lb-sec[a]
$\beta_{1,2}, \beta_{3,4}$	$= 22, 38°$[b]
$D_{1,2,3,4}$	$= 1.3, 2.0, 1.7, 1.9$, ft-lb-sec
$\tau_{1,2,3,4}$	$= 4.0, -3.9, -6.8, -6.6$, ft-lb $\times 10^{-4}$
ω_0	$= 0.0011$ rad/sec, $i = 75°$
a_x, a_y, a_z	$= 950, -500, 0$, ft^3
m_x	$= 5.1 \times 10^{-4}$ ft-lb/oersted $= 6900$ UPC
m_y	$= 0.9 \times 10^{-4}$ ft-lb/oersted $= 1200$ UPC
m_z	$= 5.2 \times 10^{-4}$ ft-lb/oersted $= 7000$ UPC
M_{s_x}	$= [6.8 - 27\, c\omega_0 t - 1.3\, c2\omega_0 t - 16.1\, s\omega_0 t - 0.3\, s2\omega_0 t] \times 10^{-5}$ ft-lb
M_{s_y}	$= M_{s_z} = 0$

[a] Four CMG's arranged in redundant vee pairs were used.
[b] Unsymmetric vee angles β_i provided roll gyroscopic torques to align nonprincipal axes with orbital axes.

As a further example of the potential of the magnetic compensation procedure, actual flight data from Agena satellite B (Table 3) was fit to determine the vehicle parameters. The magnetic compensation software was then used to determine the magnetic moments required to minimize the attitude motion. Simulations, using both the data filter and the minimizing values of the magnetic moments, were run for a period of 30 orbits. Figure 4a shows the comparisons of the original fit with the flight data, while Fig. 4b again shows the prediction capability along with the sustained performance improvement that can be expected from magnetically compensating the spacecraft. In this case, pitch oscillation amplitudes were reduced from 1.5° to about 0.4°. Yaw oscillations were reduced to less than 0.2° while roll is left essentially unchanged. The bias between simulated and actual yaw data is present since no yaw sensor was available for this flight.

Conclusions

For a class of gravity-stabilized spacecraft, capability of determining inflight magnetic moments and fitting attitude librations to mean errors less than 0.1°, using a digital simulation, has been demonstrated. The modeling accuracy for the satellite and disturbance environment has been verified by predicting satellite attitude to within 0.5° mean error over a 120 orbit span.

By proper on-orbit adjustment of compensating magnetic moments, magnetic disturbance torques can be used to control and significantly reduce attitude librations. Specifically, torques of arbitrary phase and amplitude may be generated to control pitch disturbances; effective roll and yaw control is obtained, through limited constraint on pitch control, for torques produced by magnetic moments or for torques whose phase is similar to those induced by magnetic moments. Unlike pitch, control of arbitrary disturbances in these axes can only be partially successful. For the cases considered, pitch motions have been reduced by a factor of two or more, and attitude oscillations limited to 0.5–1.0 about fixed biases for all axes. The fixed biases are known, however, through the attitude determination facility afforded by the simulation. Compensation can be effective, without magnet readjustments, for periods ranging from several weeks to months.

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A Magnetic Attitude Control System for an Axisymmetric Spinning Spacecraft

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This paper describes a theoretical study of a continuous magnetic attitude control system which points the spacecraft spin axis normal to a highly eccentric orbit plane and maintains nearly constant spin speed. A Kalman filter is used to process sampled roll error measurements and produce estimates of yaw error and attitude rates which enable the generation of minimum energy control, active nutation damping, and fast transient response for removing pointing errors. An algebraic solution of a pointing control law which uses the error state estimate to decrease energy requirements is developed. Mechanization of both pointing and spin-speed control is obtained with three logic modes based upon spin-speed error. A Lyapunov-function-generating technique is used with averaging techniques to demonstrate stability of the resulting system operating in a fluctuating magnetic field. Dynamic performance characteristics of the controller are compared to those of a simpler system without the filter.

Introduction

A SPACECRAFT which is spin-stabilized about its maximum axis of inertia is often used for Earth orbital applications because of its inherent ability to hold a fixed spin-axis orientation. The degree of accuracy to which a spin axis can be pointed is highly dependent upon the vehicle attitude sensors available and the orbital elements.

This paper is concerned with a control system which will accurately point the spin axis of such a satellite normal to the plane of a highly eccentric orbit while sustaining the spin speed. The control system has direct application to a class of drag-free geodesy satellites,[1] but it can be used for most spining satellites requiring such orbit-plane orientation. In particular, magnetic actuation of this control is investigated. The desired torque is produced by creating a dipole on the spacecraft which interacts with the Earth's magnetic field. It is assumed that the only attitude error measurements come from horizon sensors.

Much of the previous work on magnetic controllers is related to the above control problem. Wheeler[2] investigated the use of a single dipole aligned with the spin axis for pointing control. Vrablik et al.[3] discussed using electromagnetic coils with axes perpendicular to the spin axis of the LES 4 satellite for maintaining the spin axis nearly normal to a circular, equatorial orbit. Fischell[4] recognized that magnetic control could be used for regulating the spin speed of the satellite. Sonnabend[5] described a controller for keeping the spin axis of a dual-spin satellite normal to the plane of a circular orbit. The first operational magnetically controlled spinning spacecraft, the TIROS satellites, were discussed by Hecht and Manger.[6] This paper combines and extends these results to a system with limited attitude determination capability operating in an eccentric orbit.

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Coordinate Systems and Equations of Motion

Figure 1 illustrates the local reference frame of the spacecraft which has unit vector $\hat{x}_l$ oriented outward along the radius vector from the geocenter. The unit vector $\hat{z}_l$ is normal to the orbit plane and positively oriented toward the northern hemisphere, and $\hat{y}_l$ completes the right hand set. The angles i, Ω, ω_p, and f are the inclination, right ascension,